MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

Several questions on this paper required clear verbal explanations. One thing candidates could improve on for the future is to make sure their intended meaning and each stage in their workings are made clear.

General comments

When a question requires more than one answer candidates are encouraged to look for ways in which additional solutions may be generated.

Comments on specific questions

Question 1

The understanding of the requirement to integrate $f'(x)$ and the skills to do so were evident in most answers. The correct calculation of the constant of integration was also seen extensively. Whist denominators containing fractions are given credit in the working for the integration and the constant it is expected that the final expression of y or $f(x)$ will have no fractions in any denominators.

Question 2

The elimination of y to obtain a quadratic equation in x and the calculation of the discriminant was the only method seen in this question and it was often used very successfully. The solution of the resulting inequality in c was challenging for some candidates but where a sketch was used the correct regions were nearly always selected.

Question 3

(a) Most candidates were able to select the required term and then evaluate it. Occasionally the use of brackets around the variable terms was a source of errors. The selection of the required term should be encouraged rather than generating a complete expansion.

(b) The need to consider the answer to **part (a)** and the term in x^{-3} was seen in many responses and those who didn't make sign errors often reached the correct final answer.

Question 4

The two equations linking the given terms were nearly always quoted correctly. The algebra involved in the elimination from these of r or a and d was completed correctly in the better answers. Sign and algebraic errors were seen in a considerable number of answers, and these made it challenging for candidates to be able to use the arithmetic progression sum formula even though they appreciated this was required.

Question 5

(a) The required form was often quoted correctly but sometimes confused with the form $2(x-a)^2+b$. However, most answers gained some credit. An area candidates can improve on is reducing the amount of working they do as this will help them reach the right answer more easily.

(b) Many candidates were able to clearly express a correct sequence of transformations. Most candidates showed a clear description of translations using vectors and the use of a scale factor and direction with the stretch.

Question 6

(a) Most candidates were able to substitute the linear equation into the circle equation and went on to reach a correct quadratic equation in x. When this was solved using an acceptable method (factorisation, formula or completing the square) full marks were commonly gained.

Candidates should note that a clear method must be shown for solving a quadratic if full marks are to be awarded.

(b) A variety of methods were used for finding the equation of the circle. A popular successful method was to find the centre of the circle and the midpoint of AB and equate the distance between them to the radius of the required circle. Candidates should check their work carefully as sign errors were common in finding the distance between the two points. Equally successful were those who let the equation of the circle be $(x + 1)^2 + (y - 2)^2 = r^2$ and solved this with the equation of AB and realising that the resulting equation had equal roots, set the discriminant to zero to find r^2. The straightforward method of finding the radius as the distance of the centre from the line AB using a formula was rarely used but was usually successful.

Question 7

(a) Obtaining an expression with a common denominator was well understood as was expanding brackets. As in other questions careful checking of working would have enabled candidates to spot the common sign errors. Use of $\cos^2\theta + \sin^2\theta = 1$ was well understood and most candidates who were able to simplify the numerator of their single fraction went on to reach the correct expression. As with any question where the answer is given it is expected that candidates will show clearly each step in their working.

(b) Most answers showed the result of **part (a)** equated to 5 and most of these reached a correct value of $\cos\theta$. A minority found both values of $\cos\theta$ and both values of θ. It should be noted that the units of the answer should be the same as the units of the given domain.

Question 8

(a) The algebraic requirements for this part were well understood with a majority of candidates answering this question correctly. A few errors such as taking a negative square root or interchanging the coordinates of A and B could have been avoided if the given diagram had been used more effectively.

(b) Most candidates rearranged the circle equation and set up a correct integral. The integration was mostly carried out successfully. Finding the volume of the cylinder where it was seen was generally done successfully either using the volume of a cylinder formula or by integration. The substitution of the upper and lower limits from **part (a)** had to be shown for full marks to be awarded.

Question 9

(a) Those candidates who followed the guidance for this exam and showed their methods clearly were able to gain all marks either by squaring $4x^{\frac{1}{2}} = 1 + x$ and solving the resulting quadratic equation or by solving $x - 4x^{\frac{1}{2}} + 1 = 0$ as a quadratic in $x^{\frac{1}{2}}$ and squaring the two answers. Candidates need to understand that $2 - \sqrt{3}$ was in the domain and knowing this would have enabled them to find the second x value.

(b) The required composite function was nearly always found correctly and usually expanded and equated to f(x) correctly. Only the better answers showed this expression treated as an identity and the correct values of m and n appropriately selected. Candidates who had solved **part (a)** by first writing f(x) in completed square form were able to quickly compare this with their initial form of the composite function to find m and n correctly.

Question 10

(a) The trigonometry skills needed to find the required angles were evident in most answers. Candidates worked in angles or degrees to find the arc lengths correctly. The 5, 12, 13 triangle was often recognised and DE quoted correctly. Some candidates angles found to two decimal places which in this case led to acceptable answers for the perimeter. This will not always be the case and working should always be carried out using one more level of accuracy than the final answer requires.

(b) This proved to be straightforward for those who had correct angles from **part (a)** and were able to use the sector area formula. In this part angles rounded to two decimal places did not lead to an area in the required range.

Question 11

(a) The requirement to integrate y was apparent to all but the lowest scoring candidates and many error free first derivatives were seen. Setting this derivative to zero was often seen but algebraic errors resulted in few completely correct answers. Some candidates expanded $(3x-k)^2$ too early or didn't include the negative square root. Avoiding these errors would have also enabled them to answer the question successfully.

(b) Most candidates chose to differentiate f(x) and solve f′(x) = 0 rather than using k = 4 with their answer to **part (a)** even when it was available. The use of the second derivative was usually seen although only the best answers showed a correct process and clear explanation of the nature of the turning point. An area candidates can improve on is making their workings out, methods and explanations clearer.

(c) Here again the need to use the first derivative was seen in most answers. The justification of the first derivative always being positive had to be shown through reference to the nature of the terms within the derivative rather than the substitution of values into the derivative.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

Candidates need to:

- Keep in mind that Argand diagrams require the same scale on both axes, otherwise any resulting sketch is meaningless, as in **Question 2**
- Ensure they know how to work with a plot of ln*y* against ln*x*, as in **Question 3**
- Apply the chain rule in differentiation, as in **Question 4** and **Question 11(a)**
- Equate real and imaginary values in a given equation, as in **Question 6**
- Check if **parts (a)** and **(b)** of a question are linked, as in **Question 8**.

General comments

The standard of responses on this question paper was quite varied. Many candidates applied their knowledge and skills well, but others needed to show all their working which would have enabled them to perform better. For example, on Question 4, there was a given answer and candidates must ensure that they show all their working for such a question.

Candidates should ensure that they read the question carefully and try to fulfil all of its requirements especially where they are asked to give their answer in an exact form, as in **Question 9** and **Question 11(b)**. In **Question 9** the candidates are tested on their knowledge of the laws of logarithms as the question requests the answer to be in an exact form, as opposed to simply using a calculator. Likewise in **Question 11(b)** the examiner is requesting an exact form to ensure that candidates know the exact value of sine or cosine of $\frac{\pi}{4}$ as opposed to its numerical value obtained from the calculator.

Candidates are reminded to write clearly, with letters and symbols of a reasonable size, and work logically down the page to help the reader to understand their solutions.

Comments on specific questions

Question 1

Most candidates opted to solve the quadratic equation as opposed to pairs of simultaneous equations. However regardless of the approach the critical values were usually correct. Many candidates chose the incorrect region in their final answer or decided that $-\frac{9}{5}$ was less than –3.

Question 2

Many candidates incorrectly stated their centre as one of (–2, –3), (2, 3) or (2, –3). Although the radius of the correctly centred circle was usually denoted to be 2 and usually went through the point (0, 3), in many cases it did not also pass through, or close to, the points (–2, 5), (–2, 1) and (–4, 3). The half line was often a full line or did not bisect the 90° angle at the origin in the second quadrant, in fact it often passed through the centre of the circle itself. Even candidates with all the required features correct often shaded the region inside the circle to be that below the half-line, see **Key messages**.

Question 3

Candidates generally found this question challenging but many were able to take logarithms of the given equation. Common errors occurred when candidates either substituted ln$x$ and ln$y$ into the given equation, for example used (0.31, 1.21), instead of the exponential values from the given coordinates. Some also substituted ln(ln$x$) and ln(ln$y$) into the simplified equation found by taking logarithms of the given equation, for example using (ln(0.31), ln(1.21)), rather than (0.31, 1.21), see **Key messages**.

Question 4

Candidates generally found this question to be challenging, a common error was incorrect chain rule application. Some candidates confused θ with x in sin θ and dθ, whilst others replaced $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $\frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}\theta}{\mathrm{d}x}$ and then went straight from ($\cos\theta - 1$) to $-2\sin^2\left(\frac{\theta}{2}\right)$ without showing any working, see **Key messages**.

Question 5

Many candidates found this question rather straightforward and produced some excellent responses were seen. Candidates need to ensure they know the correct expansion formula for $\tan(\alpha + \beta)$ and are reminded to double check their working out, as arithmetic errors were common. Candidates who commonly scored 5 marks did not obtain the obtuse angle required.

Question 6

Most candidates successfully identified that they needed to introduce $w = x + \mathrm{i}y$ and $w^* = x - \mathrm{i}y$ and this enabled them to complete the question successfully. Candidates who tried to work with w and w^* found it more difficult to do so. However, when equating real and imaginary parts candidates needed to understand that the imaginary part of 1 was 0, not 1. That meant the equation arising from this incorrect equating of imaginary parts could not be solved. Those candidates that did realise that the imaginary part of the right side of the equation was 0 usually were able to establish one of the roots. The other root often disappeared from x being cancelled out as opposed to being set to zero. Few candidates included $w = 2 - \mathrm{i}$, having been given that real(w) $\leqslant 0$, see **Key messages**.

Question 7

(a) Candidates need to bear in mind that sketching and plotting graphs are different processes. Most candidates just plotted around 4 points for each graph and joined these points. Without graph paper and an accurate scale it is impossible to obtain a smooth curve using this approach. Candidates need to be aware that sketched graphs should show the main features of the functions over nearly the whole interval. So $y_1 = 4 - x^2$ needs to be a smooth curve passing through (0, 4) and (2, 0), with an increasing negative gradient, and reaching $x = \pi$ or close to that value. Whereas $y_2 = \sec\frac{1}{2}x$ should pass through (0, 1), have an increasing gradient and have a large value of y as x approaches π. In addition the point of intersection needs to be related to the root by some means or other, usually via being highlighted or some comment at the side of the graphs.

(b) Candidates needed to understand that the values 1 and 2 are not in degrees, knowing this would have helped them to answer this question successfully. They should have been comparing the values of y_1 and y_2 at these two values of x (using radians) or comparing $y_2 - y_1$ with zero. However, it should be noted that there are many other functions that can be used at these values, for example x and $\left(4 - \sec\frac{1}{2}x\right)^{\frac{1}{2}}$

(c) Many candidates used degrees to answer this question, therefore were unable to get the correct result.

Question 8

(a) Many candidates answered this question well. Occasionally candidates made arithmetic errors towards the end of their long division.

(b) Candidates needed to be aware of the link between **parts (a)** and **(b)** and doing so would have helped them answer this question more successfully. Some candidates identified the relationship between the two parts but were unable to express the integrand as $\text{Quotient} + \dfrac{\text{Remainder}}{4x^2+1}$, often omitting the denominator and finishing with the integrand $2x + 7$. Other candidates had the quotient divided by $(4x^2 + 1)$. This was all before reaching the integration stage. When integration did take place, it usually resulted in the quotient being integrated correctly but the rest either being a ln term, or a multiple of $\tan^{-1}(2x)$ with an incorrect coefficient. Some candidates who reached an expression of the form $ax^2 + bx + k\tan^{-1}(2x)$ but they needed to return their $\tan^{-1}(1)$ as $\dfrac{\pi}{4}$ and not 45°, see **Key messages**.

Question 9

Most candidates separated correctly and integrated the term $\dfrac{1}{y}$ for the first two marks. Others managed to introduce partial fractions to convert the integrand of the function of x into a form that they could integrate. Most of these candidates were successful with the integration of $\dfrac{1}{x+1}$ but struggled with the coefficient when integrating $\dfrac{1}{3x+1}$. Evaluating the value of the constant of integration and determining $y(3)$, where exact values were required, usually saw candidates turning to decimals when this was not required, see **Key messages**.

Question 10

(a) Most candidates could establish vector ***AB*** correctly, although some candidates thought that this was the vector equation of the line. Many candidates used the correct method for the vector equation of the line, however the left hand side of this equation was commonly either expressed in words, or vector ***AB*** or denoted by l_{AB} instead of the vector **r**.

(b) Correct angles were often found by those candidates commencing with the correct directions of ***AB*** and *l*. Several candidates had at least one of these vectors incorrect as they used either a point on line *l* or ***OA*** or ***OB***.

(c) Nearly all candidates correctly established a general point on the line *AB* or *l* in component form and solved for one of the line parameters. Some candidates, having found the parameter correctly, made a mistake in trying to show that the coordinates of the points of intersection were different for the two lines. It should be stated that there are many different approaches by which this question can be solved by using any pair of equations from the available three equations. In addition candidates can compare points of intersection or different values of a parameter found using different pairs of equations.

Question 11

(a) Most candidates knew how to proceed with this question but experienced problems applying the chain rule to cos2*x*. Although this problem could be made much simpler by substituting for cos2*x* in terms of $\cos^2 x$ this is not usually the case and it is advisable to just differentiate the given expression, as such substitutions often lead to other problems. Despite an incorrect differentiation using the chain rule, provided the error was restricted to the omission of the factor 2, it was possible to equate the derivative to zero following one or two substitutions of the double angle formulae and establish an equation that was acceptable for the method marks. 2sin2*x* was often replaced by 2sin*x*cos*x* making it very difficult to know if the double angle substitution was correct. It is essential that the initial differentiation is undertaken virtually correct otherwise it is impossible to establish a correct trigonometrical equation that is solvable.

(b) Many candidates struggled to establish a correct integral in terms of *u*, however many good attempts were seen. A common error was the inaccurate replacement of cos2*x* by $2\cos^2 x - 1$ accurately. If candidates established the correct integral in *u* they usually completed the question successfully, although occasionally the *x* limits were muddled with the *u* limits or they were reversed, see **Key messages**.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures or angles correct to one decimal place as stated on the question paper (See **Question 3**, **Question 4** and **Question 5)**. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 3**, **Question 4** and in **Question 7**.
- In questions such as **Question 6** in this paper, where acceleration is given as a function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

There were some excellent responses which demonstrated clear method. Overall a wide range of performance was seen but the questions were generally well answered.

Some candidates did not give answers to 3 significant figures as requested and prematurely approximated within their calculations leading to the final answer. This was often seen in **Questions 3**, **4** and **5**. In **Question 4** and **Question 7** the sine of an angle is given in the question. In such questions it was not necessary to determine the actual angle to 1 decimal place as this often leads to premature approximation and frequently also to a loss of accuracy marks. In fact if this exact angle is not used then it is not possible to achieve the correct given answer in **Question 7(a)**.

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer, such as in this paper in **Question 7(a)**, unless this value is used.

Comments on specific questions

Question 1

(a) The majority of candidates answered this part well. It was necessary to find the potential energy gained in the process of raising the block by 15 m. In addition to this the given work done against resistance must be added. Most candidates correctly found the potential energy. A common error was to subtract the given work done against resistance instead of adding it.

(b) Many candidates correctly attempted to find the required time by using the definition of average power in the form $\text{Average power} = \dfrac{\text{Work done}}{\text{Time}}$. Some candidates preferred to use the relationship $P = Fv$ but this required finding the average force acting as $F = \dfrac{\text{Total Work Done}}{15}$ and the average velocity $v = \dfrac{15}{t}$ needed to travel 15 m in t seconds. Either method was acceptable.

Question 2

(a) In this question there were several different approaches that could be taken but all of these used one or more of the equations for constant acceleration. The most straightforward approach which was adopted by the majority of candidates was to use the equation $v^2 = u^2 + 2as$ with $v = 0$, $a = -g$ and $s = 20$. Care must be taken when using this equation, if the upwards direction is taken as positive, to ensure the correct sign of the acceleration. This error led to solutions in which some candidates' value of u^2 was negative. Overall most candidates found this result correctly.

(b) In this part of the question there were several different possible approaches. A method used by many candidates was to use the equation $s = ut + \frac{1}{2}at^2$ with $s = 15$, $u = 20$, $a = -g$ and solve this to find the two values of t when the particle is at 15 m above the ground. By subtracting the two values the required time that the particle is at least 15 m above the ground can be found. Most candidates found the correct value. Some added the two t values instead of subtracting. Others used the incorrect sign for the acceleration.

Question 3

(a) In this question, since the car-trailer system is travelling at constant speed, the problem can be solved either in terms of a work/energy equation or in terms of the forces acting on the system. Those who chose to use forces had to determine the constant frictional force as $\frac{40\,000}{50}$. As the system moves with constant speed, the net force acting parallel to the hill must be zero. This means that the component of the weight of the car-trailer system down the plane is equal to the resistance force. Solving this equation gives the required value of m. Alternatively an energy approach could be taken in which the loss of potential energy is exactly balanced by the given work done against resistances. Most candidates performed well on this question. Some lost accuracy in their final answer by prematurely approximating their calculations.

(b) In this part of the question either the car or the trailer must be considered separately since the tension in the tow-bar is required. The most straightforward approach is to apply Newton's second law to the trailer using $a = 0$. If the force in the tow-bar is treated as a tension then the forces acting down the hill are the required tension and the component of the weight of the trailer. These two forces are balanced by the resistance force on the trailer of 200 N. Equating these forces gives the required tension. Candidates found this part to be challenging, often as they did not consider the component of the weight of the trailer.

Question 4

(a) This question was well done by the majority of candidates. The problem involved applying Newton's second law to the motion of the cyclist and this involved four terms. First it is necessary to determine the driving force, DF, by using the formula $\text{DF} = \frac{P}{v} = \frac{180}{6}$. The forces acting on the cyclist are the driving force, the component of the weight of the cyclist and bicycle down the hill and the required resistance force F. This combination of forces is equated to mass $\times$ acceleration and this can then be solved for F. Candidates needed to ensure they used the correct component of the weight, check there were no sign errors and not leave out force terms. This would have helped them to answer the question successfully.

(b) In this part there is no acceleration as it is required to find the steady speed. The forces acting are the new driving force $\frac{180}{v}$, the component of the weight down the hill and the resistance force found in **4(a)**. The driving force is balanced by the sum of the resistance force and the component of the weight and this equation leads to the required value of $\boldsymbol{v}$. Some candidates continued to use the 30 N driving force found in **4(a)**. Other common errors generally involved incorrect signs and trigonometric errors.

Question 5

(a) Overall, candidates made a good attempt at this question. The method used by most candidates was to resolve the given forces horizontally and vertically and equate both of these to zero since the forces are in equilibrium. At this stage this gives two simultaneous equations in ***F*** and ***G*** which must be solved. The majority of errors that were seen were numerical errors involved in the solution. Several candidates prematurely approximated during their calculations and so lost the accuracy of their final answer.

(b) In this part of the question many candidates misunderstood the wording. It is given that the resultant force is perpendicular to the 10 N force. This means that there is a zero component in the vertical direction (the direction of the 10 N). Hence the sum of the components of the forces parallel to the direction of the 10 N force must be equated to zero and this gives the required value of ***G***. However, many candidates incorrectly equated the sum of the horizontal components of the forces to zero.

Question 6

(a) Many good responses to this question part were seen. Since the acceleration is given as a function of t the constant acceleration equations cannot be applied and that calculus techniques are required. The value of k that is required is the time at which the particle is at rest and so it is necessary to find the velocity v by integrating the given acceleration and setting this to zero. The majority of candidates took this approach. The most common errors were in solving the resulting equation, with some candidates having difficulty with the fractional powers.

(b) In this part of the question it is necessary to determine when the particle reaches its maximum velocity. This occurs when the acceleration is zero and so the given expression for a must be set to zero and solved to find the time t at which the particle reaches its maximum velocity. Once this value of t has been found it can be substituted into the expression for v found in **6(a)** which enables the required velocity to be found. Most candidates performed well on this part. The majority of the errors seen were in solving the equation $a = 0$ due to the challenge of dealing with fractional powers.

(c) In this part an expression for the displacement is required. This can be found by integrating the expression found for velocity in **6(a)**. Most candidates made a good attempt at this integration. Once this expression is found it is necessary to use limits on the integral. The upper limit is the value of k found in **6(a)** and the lower limit is the value of t found in **6(b)**. It is important to make sure that these upper and lower limits are used correctly.

Question 7

(a) This question involves an exact given answer. In order to achieve this value correctly use must be made of the given exact angle property but the angle itself must not be approximated. There are two possible approaches, either a work/energy method or one involving acceleration and Newton's Laws. Either method is acceptable. If the Newton's law approach is taken, firstly the acceleration must be found from the given information. Newton's law can then be applied along the wire using this acceleration to determine the friction force, F. Resolving forces perpendicular to the plane will give a value for the normal reaction, R. Use can then be made of the relationship $\mu = \frac{F}{R}$ in order to prove that the value of $\mu = 0.25$. Most candidates used this method and many good solutions were seen. Some candidates needed to take care when making calculations and to ensure they were confident using the right symbols and terms for Newton's law equation.

(b) In this part it is first necessary to apply the principle of conservation of momentum to the two particles so that the initial speed of particle B can be found. Newton's law must then be applied to particle B to determine its acceleration from the given information relating to the coefficient of friction between B and the wire. The acceleration of A is the same as in **7(a)**. Since the initial velocities and the accelerations of both particles are known, it is now possible to find expressions for the displacements of A and B after the collision. Equating these two expressions gives the time at which A and B collide again. Most candidates applied the principle of conservation of momentum correctly. However, many continued to use the displacement 0.45 m which did not have any involvement in this part. Errors were commonly seen in finding the acceleration of B.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. When errors are corrected, candidates would be well advised to cross through and replace the term, rather than overwriting.

Candidates should state only non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure value. The only exception is if a value is stated within the question. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

Candidates should be aware that different skills and techniques can be included in the same question part, for example, **Question 2(b)**, and to be confident in using work that has already been completed in later question parts, for example **Question 4(c)** could efficiently utilise work from **4(a)** and **4(b)**.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that histograms are a visual representation of the distribution and should be constructed accurately. It was encouraging that the labelling of the statistical diagram has improved.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates did not complete the last question. A few candidates did not appear to have prepared well for the assessment, with little or no progress made in many questions. Many good solutions were seen for **Questions 1** and **4**. The context in **Questions 2**, **5** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

(a) Almost all candidates attempted a probability distribution table. The best solutions included an outcome space diagram which was an efficient method to identify the required probabilities. Weaker solutions often struggled with the negative numbers and contained arithmetical errors. A small number of solutions omitted 0 as an outcome. In general, probabilities were given as fractions but some candidates converted to decimals and then rounded to 3 significant figures, which resulted in a total probability greater than 1. Where a calculation approach was used, most solutions determined the final probability by subtracting from 1.

(b) Most candidates used the standard variance formula accurately with the values from the probability distribution table in **1(a)**. The best solutions provided a fully unsimplified expression while weaker solutions stated the values to be summed. A small number of solutions also calculated E(X) rather than use the value provided.

Very few solutions used the more numerically complex $\sum \mathrm{P}(X)(X-\mathrm{E}(X))^2$ approach and when used there were often arithmetical slips in the work.

Question 2

(a) This binomial approximation question was answered well by many candidates. The best solutions stated the unsimplified expression and then calculated the final answer using the calculator efficiently. Misinterpreting the success criteria was the most common error with many answers including two days in the calculated probability. The interpretation of success criteria is an essential skill in this component, and something that candidates would be well advised to practice.

A small number of candidates worked to 3 significant figures throughout, and this premature approximation did not lead to a value within the acceptable range.

(b) Most candidates found this two-stage problem extremely challenging. The best solutions refined the method from **2(a)** with the new success criteria and then stated an unsimplified binomial term for two of the three periods fulfilling the given requirement. Many solutions were incomplete, either simply attempting to find the probability of 0 out of 7 days with more than 10 cm of rain and some also tried calculating the probability of 0 out of 21 days with more than 10 cm of rain.

Question 3

The quality of histograms presented in this question was variable, with many being drawn without the use of a ruler, which is unlikely to provide the expected level of accuracy. Most diagrams used scales that were reasonable and sensible to achieve accurate plots from their values.

(a) The best solutions stated the class widths before calculating the frequency density, used scales which enabled values to be plotted accurately, often translating the time axis so that the continuity adjustment was on a grid line, used a ruler for lines and labelled both axes appropriately. A common error was to assume the upper boundary of the class was stated in the data table, which led to incorrect frequency densities and placement of the class boundaries on the histogram. It was encouraging that the inclusion of units in the time axis label was more common.
The weakest solutions simply constructed a bar chart from the data table.

(b) Although a large number of candidates identified the correct class containing the median value, the best solutions included justification for the decision. A common error was to simply state the middle class from the data table without justification.

(c) A significant number of candidates made no attempt at this part. Candidates who attempted this question usually correctly referenced that the data was skewed, although better responses tried to give an orientation to the skew which was not required and there was some confusion about the appropriate description (left or negative skew would be anticipated). Some candidates identified that the data was not symmetrical. Many comments referred to outliers or extreme values, often in a generic manner, which was not relevant in this context.

Question 4

Almost all candidates recognised that the question required the normal approximation throughout, and that the variable was continuous and did not require a continuity correction to be used in the standardisation formula. The best solutions often contained a sketch of the normal distribution curve with the required probability area highlighted.

(a) Almost all candidates substituted accurately at least once into the standardisation formula. A variety of different approaches for obtaining the required probability area were seen, with the best solutions providing clear supporting justification for the calculation. A noticeable number of

solutions were inaccurate due to premature approximation in the process and $\Phi(1.16)$ being found. A common error was to simply find the difference between $\Phi(1.167)$ and $\Phi(1.5)$.
A small number of candidates assumed that 46 and 62 were equidistant from the mean.

(b) The best solutions stated the appropriate critical *z*-value, formed an equation with the standardisation formula and solved with clear supporting algebra. Candidates should note that the use of critical values from the tables is an expectation within the component and alternative values will not gain full credit. A number of solutions which found a *z*-value subtracted from 1 prior to forming the equation while others equated a probability with the standardisation formula.

(c) A significant number of candidates made no attempt at this question and many found it very challenging. The best solutions recognised that the question built upon the previous parts, using their work in **4(a)** to state P(male < 46) and the standard deviation from **4(b)** to calculate P(female < 46). Weaker solutions often recalculated P(male < 46) and used the male standard deviation to calculate P(female < 46). The majority of candidates who obtained two probabilities did find the product as required, although the common error was finding their sum, although the difference was also seen if previous errors resulted in a sum greater than 1.

Question 5

Most candidates identified that the use of combinations and permutations was required in the question and that **part (c)** required a different approach to the other parts. The use of simple diagrams to visualise the given criteria was often seen in good solutions.

(a) Most solutions recognised that this was a fairly standard selection question involving combinations. The most common approach was to combine the boys and girls into a single group to choose 4 from, although candidates who considered all the scenarios that the boys and girls could be selected to form a group were normally successful. Many solutions did not identify that there were five different adults that could join the group and omitted to multiply by 5. A small number of solutions added rather than multiplied by this value.

(b) A large number of candidates recognised the structure of the success criteria and identified the possible scenarios initially. The best solutions used a systematic approach when listing, stated unsimplified calculations for each scenario before summing to obtain a total. Common errors were to either only calculate the number of ways the adults could be selected in each scenario or assume that the boys and girls not selected for the scenario were included with the adults. The omission of possible scenarios often occurred when a less systematic approach to listing was taken.

Candidates should be aware that clear communication of their method is expected, and in questions like this linking calculations to scenarios should be explicit.

A number of responses unsuccessfully attempted to use an approach where two boys and one girl are selected initially and then the remaining two members of the group would be selected from the nine people but did not consider where selections are repeated.

(c) Many found this question very challenging. The best solutions nearly always had a simple diagram that visualised the requirements of the question. The diagram could guide directly to the required calculation for the number of arrangements. Weaker solutions either used combinations to select the two adults for the ends or simply assumed that there was only one way to select these adults. A common error was to calculate the number of arrangements that the three boys could stand in. Several correct unsimplified expressions were evaluated inaccurately.

Question 6

Some excellent solutions were seen for the question as a whole. These frequently included a tree diagram to assist with **parts (c)** to **(e)**, clear communication of the success criteria in **parts (a)** and **(b)** and accurate evaluation of probabilities using fractions.

(a) The majority of candidates identified that a geometric approximation was appropriate. However, many did not interpret the success criteria accurately and calculated either the sixth or eighth chocolate chosen would have a lemon flavour. Weaker solutions simply calculated the probability of not choosing six lemon flavour chocolates. Candidates should be aware that the interpretation of success criteria is essential in this component.

(b) A significant number of candidates did not attempt this question. The best solutions recognised that choosing a lemon flavour chocolate after checking at least six chocolates has the same probability as not choosing a lemon chocolate six times. The alternative approach of finding the complement of the probability of choosing a lemon flavour on either the first, second, … and sixth selection was usually successful when attempted. A small number of candidates used a binomial approximation approach without success.

(c) Most candidates recognised that there was a new scenario for this part.

This was quite a straightforward without replacement probability context, with the best solutions including some justification for the calculation presented identifying that there were initially 10 chocolates which were not flavoured orange. Many candidates presented a tree diagram from the given data, and so identified all the different scenarios that lemon and strawberry flavour chocolates could be chosen. These individual probabilities were calculated and then totalled. This more involved approach often involved the omission of one or more possible scenarios. The weakest attempts often replaced the chosen chocolate or removed the chocolate without reducing the overall number of chocolates in the box.

(d) Candidates who had drawn a tree diagram in **part (c)** were often successful in this part, supporting their solution with outcomes identified on the tree diagram. Almost all solutions presented did use the correct probabilities without replacement. The most common error was not recognising that there were different orders that the chocolates could be selected in and omitting to multiply by 6.

(e) Many candidates found this question challenging and a significant number of candidates made little or no progress to a solution. Better solutions stated the general conditional probability formula, then calculated the values for the numerator and recognised that the denominator was the answer of **part (c)**. A common error was not recognising that there were different ways that one lemon and two strawberry chocolates could be selected.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

- Candidates are advised to always read the question carefully.
- All answers should be fully supported with the relevant working.
- When carrying out a significance test the comparison between the test value and critical value (or equivalent comparison) must be clearly shown in order to justify the conclusion drawn.
- When writing conclusions to Hypotheses tests the answer must be given in context and there must be a level of uncertainty in the language used.
- Candidates need to be able to recognise the appropriate distribution (e.g. binomial) in a given scenario.

General comments

This was a reasonably well attempted paper, with most candidates finding questions to demonstrate their knowledge, though there were questions which candidates found challenging.

The questions on the paper which candidates found particularly demanding were **Questions 5(b)** and **6(d)**. Other questions that some candidates found demanding were **1(b)**, **6(a)** and **Question 3** whilst the questions that many candidates found reasonably straightforward were **Questions 4**, **5(a)**, **6(b)** and **7(a)(ii)**.

Timing did not appear to be a problem for candidates.

The comments below indicate common errors and misconceptions, however, there were also some full and correct solutions presented too.

Comments on specific questions

Question 1

(a) In general this question was reasonably well attempted, though there were the usual cases where candidates confused the two formulae for the unbiased estimate of the variance. Very few candidates calculated the biased estimate and calculations to find Σx and Σx^2 were mainly accurate, though there was some confusion between Σx^2 and $(\Sigma x)^2$. Some candidates did not maintain 3 significant figure accuracy due to premature rounding.

(b) Many candidates did not identify that the sample size was small and made comments such as 'the data items collected were too similar/not varied enough' or 'not measured accurately enough'. Some candidates suggested the whole population needed to be used rather than a sample.

Question 2

Many candidates found this question to be challenging. It is important for candidates to be able to recognise the distribution in a given scenario (here $B\left(300, \frac{1}{5}\right)$) and to be able to find a suitable approximating distribution (here $N(60,48)$). Many candidates used an incorrect normal distribution and use of a wrong, or no, continuity correction was commonly seen. It was important that a clear and valid comparison was shown between the calculated z and the critical z, or equivalent area comparison (where a common error seen was to compare with 0.05 rather than 0.025). The conclusion reached must be in context, using non-definite language; many candidates merely left their conclusion as 'Reject H_0' which without further in-context and non-definite comment (such as 'there is evidence that the spinner is biased') this was not sufficient.

Question 3

Candidates also found this question to be challenging. Candidates who successfully realised and used est(p) = 0.4 were usually successful in at least finding the z value (2.054), and some went on to correctly find the confidence level of 96 per cent (others incorrectly thought it to be 98 per cent). Many candidates could not progress with the question without a value for p whilst others incorrectly used 0.445 or 0.355 for p.

Question 4

(a) On this question many candidates correctly declared their hypotheses, standardised to find the required z value and usually gave a clear and valid comparison in order to make their conclusion. As in **Question 2**, it was important to give the conclusion in context and to use language with the required level of uncertainty. On the whole candidates were confident in their approach to this question.

(b) Many candidates appreciated that they had to check if H_0 had been accepted or rejected in order to answer the question. Some candidates showed confusion between accepting or rejecting a hypothesis and it actually being true or false.

Question 5

(a) This part was confidently answered. Use of N(91.5,31.25) was often seen with many candidates reaching the required answer. Common errors included finding an incorrect value for the variance and calculated the wrong probability area (> 0.5 rather than < 0.5).

(b) Many candidates found this question part to be particularly challenging. Many correctly found E(Difference)=0 but commonly errors were made in calculating the variance. Standardising was often done well, but not all candidates fully realised what the question was asking for. In order to calculate the probability that the difference in heights between two randomly chosen buildings was less than 1, the height of (Building_1 – Building_2) < 1 needed to be considered as well as the height of (Building_2 – Building_1) < 1.

Question 6

(a) Candidates needed to be aware that here the probability density function was a quadratic curve and that the symmetry of the curve meant that the mean was 2. Many candidates did not mention symmetry and thought that the mean was 2 because the mean of the end points 1 and 3 was 2 with $\frac{1+3}{2}$ often being given as their answer. Others stated 3 – 1 = 2 as an incorrect reason for the mean to be 2.

(b) A large number of candidates were able to correctly show that k was $\frac{3}{4}$, mostly with sufficient working provided. Candidates mainly used the fact that the area between 1 and 3 was equal to 1, but some correctly used the fact that the mean was equal to 2 in order to find k.

(c) A reasonable attempt was made here to find the variance. Common errors included omitting to subtract the mean squared, and sign errors were common (mostly from omitting the negative sign from the function).

(d) Many candidates were unsure how to calculate the probability of greater than 2.5 and some incorrectly used a normal distribution. Those who did a correct calculation to find the probability of greater than 2.5 usually reached the correct part answer of $\frac{5}{32}$. However, most candidates though this was the final answer, or multiplied this answer by 3, and did not answer the question posed.

Question 7

(a) (i) Most candidates used the correct values of 1.2 and 1.08 in their Poisson expressions. However, few went on to combine their two answers correctly, with many adding them and others leaving them as two separate answers.

(ii) This part of the question was answered well. Many candidates used the correct value of lambda (2.28) and used a correct Poisson expression to reach the required answer. Errors included having an additional incorrect term in their Poisson expression or using an incorrect value of lambda.

(b) Some candidates successfully found that k was 2 but although some found two correct expressions in λ and μ they did not use them correctly to find k. Algebraic errors were often seen, in particular the use of $2\mu^2$ rather than $(2\mu)^2$.